Large-Scale Multi-Objective Evolutionary Optimization

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***Abstract*—With wide applications to various practical problems, multi-objective evolutionary optimization has become a popular research field in the past 20 years. However, the performance of traditional Multi-Objective Evolutionary Algorithms (MOEAs) often deteriorates rapidly as the number of decision variables increases. The specific difficulties behind this performance deterioration still remain unclear despite recent efforts to design new algorithms. In this work, the exclusive challenges along with the increase of the number of variables of a Multi-objective Optimization Problem (MOP) are examined empirically, and the popular benchmarks are categorized into three groups accordingly. Problems in the first category only require MOEAs to have stronger convergence. Problems that require MOEAs to have stronger diversification but ignore a correlation between position and distance functions are grouped as the second. The rest of the problems that pose a great challenge to the balance between diversification and convergence by considering a correlation between position and distance functions are grouped as the third. While existing large-scale MOEAs perform well on the problems in the first two categories, they suffer a significant loss when applied to those in the third category. To solve large-scale MOPs in this category, we have developed a novel indicator-based evolutionary algorithm with an enhanced diversification mechanism. The proposed algorithm incorporates a new solution generator with an external archive, thus forcing the search towards different sub-regions of the Pareto front using a dual local search mechanism. The results obtained by applying the proposed algorithm to a wide variety of problems with up to 8192 variables demonstrate that it outperforms eight state-of-the-art approaches on the examined problems in the third category and show its advantage in the balance between diversification and convergence.**

***Index Terms*—scalability, multi-objective optimization, large-scale optimization, indicator-based evolutionary algorithm**

I. Introduction

Scalability of Evolutionary Algorithms (EAs) has been a long-standing concern in the Evolutionary Computation (EC) community [1]. In the literature of multi-objective optimization, scalability with respect to the number of objectives has attracted considerable research interests [2]–[4]. However, scalability with respect to the number of decision variables remains inadequately explored. Given that the performance of most existing Multi-Objective EAs (MOEAs) deteriorates severely with the increase in the number of decision variables [5], [6], this deficiency has motivated the present study[1].

When a large number of variables is involved in an MOP, the decision space is of high dimensionality, akin to the large-scale single-objective optimization [7], [8]. Thus, some efforts have been made to design new algorithms by adapting existing techniques to large-scale single-objective optimization to the MOEA context. For example, the widely-used concept of cooperative coevolution [9] has led to the proposal of CCGDE3 [10], MOEA/D^2 [11], MOEA/D-RDG [12], MOEA/DVA [13], LMEA [14], DPCCMOEA [15] and WOF [16]–[18]; the dimensionality reduction based on random embedding [19] was used for the design of ReMO [20]. Yet, these efforts have failed to ensure that the aforementioned MOEAs scale well to a number of large-scale Multi-objective Optimization Problems (MOPs), as shown in Section II. This is not surprising, since the target of multi-objective optimization, i.e., finding the Pareto optimal set, is different from that of single-objective optimization. Thus, it is unlikely that direct application of methods aiming at large-scale single-objective problems would perform ideally in the MOP domain. Still, the new challenges introduced by large-scale MOPs have rarely been discussed in the literature.

The aim of this work is to bridge the gap in extant knowledge by first performing a comprehensive analysis of the exclusive challenges to MOEAs that tend to be more severe when the number of decision variables increases, and then proposing a new scalable MOEA for large-scale multi-objective optimization.

Empirical studies were carried out for the analysis as mathematical knowledge of problems could be hardly available in a real-world scenario. Three aspects were examined to show how increasing the number of decision variables affects the performance of existing MOEAs: efficiency (i.e., the computational effort that a given algorithm requires to reach the Pareto front), quality of the obtained solution sets, and the search behavior of a given algorithm in the objective space.

The experimental findings based on applying MOEAs to four commonly used benchmarks (including ZDT [21], DTLZ [22], WFG [23] and CEC 2009 competition [24]) indicate that different problems may pose different challenges to the algorithms when the number of variables increases. Thereby, these problems can be categorized into three groups: convergence-focused (ZDT, DTLZ and WFG), diversity-type I (WFG with varying numbers of position variables) and diversity-type II (CEC 2009 competition) problems. Specifically, the convergence-focused problems require MOEAs to exhibit sufficient convergence ability, as a set of well-distributed solutions can be obtained relatively easily once several good solutions have been found. On the other hand,

[1]Although research on the scalability of many-objective EAs also required further attention, this work focuses on two objectives to study the essential features brought out by a large number of decision variables.

when the number of decision variables in the rest of the problems increases, achieving a good spread along the Pareto front becomes more difficult, while convergence is affected relatively mildly for some existing MOEAs. Among these problems, it is further found that such a diversity loss has rarely been observed when NSGA-II is adopted, which motivates a further analysis of the design of these problems and results in the two categories of diversity-type I and II problems.

The difficulties associated with convergence-focused type can be adequately mitigated by applying the techniques employed in large-scale single-objective EAs. The recently developed large-scale optimizer WOF [17] based on this strategy is shown to be effective and efficient in solving problems in this category. The diversity loss associated with diversity-type I problems, as will be shown in Section II, is likely to be manageable by adopting NSGA-II. Thus, the WOF, which employs the same crowding distance as that of NSGA-II, could be a good alternative in this scenario. However, the performance of all evaluated algorithms when applied to the diversity-type II problems deteriorates significantly as the number of variables increases. As recent large-scale optimizers, including WOF, exhibit inferior performance to that of classical algorithms, this highlights the need for a novel large-scale MOEA with an enhanced diversification mechanism to deal with these problems.

A novel scalable MOEA with Dual Local Search (DLS-MOEA) is proposed. It is based on the SMS-EMOA framework [25], enhanced by a new solution generator. SMS-EMOA is an indicator-based algorithm with the aim of directly maximizing the hypervolume. Intuitively, SMS-EMOA should possess both high diversification and convergence, since the hypervolume is an overall performance indicator [26]. However, its actual diversification ability is inadequate, as it lacks in a mechanism by which the newly generated solution can be biased towards different parts of the Pareto front. This weakness becomes more prominent when solving large-scale MOPs, as discussed in Section II. A similar behavior is also observed when SMS-EMOA is used to solve many-objective knapsack problems [27].

Given the aforementioned shortcomings, the main aim of the proposed DLS-MOEA is to enhance the diversification ability of SMS-EMOA by exploiting a new solution generator, while inheriting the strength of SMS-EMOA with respect to convergence. The motivation behind the new solution generator is to encourage diversity by forcing the search towards different parts of the Pareto front. More specifically, it maintains an external archive during the search and, at a certain search point, solutions stored in the archive are forced to explore different parts of the non-dominated front constructed by the current archive. It might be trivial to form such a search bias when dealing with small-scale MOPs, since the search space is relatively limited. When the number of variables is large, however, the search guidance becomes much more complicated, because the search space increases rapidly and the mappings between the Pareto sets and Pareto front can be quite complex. To address this issue, a dual local search mechanism is implemented in the DLS-MOEA to guide the search in the archive.

The effectiveness of the DLS-MOEA is examined through comprehensive empirical experiments. Eight state-of-the-art algorithms were applied to the diversity-type II problems, involving 1024–8192 variables. The obtained results indicate that the proposed DLS-MOEA significantly outperforms the eight compared algorithms when applied to the examined diversity-type II problems, and provides a good balance between diversification and convergence for large-scale MOPs.

The remainder of this paper is organized as follows. In Section II, an analysis of challenges that might be more severe for MOEAs when solving large-scale MOPs is presented. Section III provides the details of the proposed DLS-MOEA. Section IV is devoted to the experimental studies in which DLS-MOEA performance is compared to that of eight state-of-the-art algorithms. The key findings and suggestions for future research directions are then provided in Section V, thus concluding this paper.

II. Challenges Associated with Large-Scale Multi-Objective Optimization

In this section, an empirical analysis is presented to identify why existing MOEAs cannot scale well to large-scale MOPs. Some theoretical analysis is also included to help deepen the understanding of the experimental results.

A. Experimental Setup

The algorithms used in the experiments include: (1) three MOEAs from the classes of Pareto-based, decomposition-based and indicator-based MOEAs, i.e., NSGA-II [28], MOEA/D [29], SMS-EMOA [25], and (2) two state-of-the-art large-scale MOEAs that employ decomposition strategy and dimensionality reduction, i.e., WOF-SMPSO [17] and Re-NSGA-II [20]. The parameter settings of the algorithms are shown in Table I, and the former three all use SBX crossover with $p_c = 0.9, \eta_c = 20$ and polynomial mutation with $p_m = 1/D, \eta_m = 20$, where D is the number of decision variables. Four well-defined benchmark suites were tested, namely ZDT [21], DTLZ [22], WFG [23] and CEC 2009 competition [24]. Specifically, on the one hand, ZDT4, DTLZ1, DTLZ3, DTLZ6, WFG1–3, WFG5–6, WFG8–9, and UF1–7 were examined, among which WFG problems were tested with conventional parameters, i.e., D variables were split into $2(M-1)$ position and $D-2(M-1)$ distance variables, where M is the number of objectives [30]. Each problem involved four instances with 1024, 2048, 4096 and 8192 variables. On the other hand, WFG1–9 with varying numbers of position variables were also examined. Each problem involved four instances with 200, 400, 600 and 800 position variables, in addition to 1000 distance variables. To be different from the former setting, these WFG problems are referred to as WFGpos problems.

To identify how the performance of the MOEAs changes with the number of variables, these algorithms were examined

TABLE I
COMMON SETTINGS AND ALGORITHM PARAMETER SETTINGS FOR WOF-SMPSO AND RE-NSGA-II

Common Settings	
Population Size	40
Stopping Conditions	1.0E+07 fitness evaluations or a solution set with its hypervolume exceeding 90% of the hypervolume of the Pareto front is obtained
Independent Runs	25
WOF-SMPSO	
Grouping Strategy	Ordered Grouping
Problem Transformation	p-Value Transformation
t1Evaluations	1000
t2Evaluations	500
Weight Optimization Population Size	10
Number of Groups	4
Number of Chosen Solutions	3
Method to Choose Q	Crowding Distance
p Value	0.2
Delta	0.5
Polynomial Mutation η_m	20
Mutation Probability	$1/D^*$
Re-NSGA-II	
Effective Dimension Bound	50
Crossover Percentage	0.7
Mutation Percentage	0.4
Mutation Probability	0.02

$^*D$ is the number of decision variables

TABLE II
SOLUTION QUALITY IN TERMS OF RELATIVE HYPERVOLUME VALUES (MEAN AND STANDARD DEVIATION) OF NSGA-II, MOEA/D, SMS-EMOA, RE-NSGA-II AND WOF-SMPSO ON DTLZ1, WFGPOS3 AND UF1. THE BEST IS HIGHLIGHTED IN GREY. THE COMPARISONS BETWEEN THE BEST AND OTHER ALGORITHMS ARE SHOWN IN ♯, ∗ AND §, WHICH INDICATE IT PERFORMED SIGNIFICANTLY WORSE, BETTER AND COMPARATIVELY THAN THE SPECIFIED ALGORITHM, RESPECTIVELY.

		NSGA-II	**MOEA/D**	**SMS-EMOA**	**Re-NSGA-II**	**WOF-SMPSO**
DTLZ1	1024	∗0.741	∗0.830	∗0.876	∗0.000	0.931
		0.028	0.017	0.009	0.000	0.154
DTLZ1	2048	∗0.000	∗0.000	∗0.002	∗0.000	0.912
		0.000	0.000	0.006	0.000	0.174
DTLZ1	4096	∗0.000	∗0.000	∗0.000	∗0.000	0.801
		0.000	0.000	0.000	0.000	0.233
DTLZ1	8192	∗0.000	∗0.000	∗0.000	∗0.000	0.745
		0.000	0.000	0.000	0.000	0.237
UF1	1024	0.876	∗0.829	∗0.828	∗0.000	∗0.567
		0.019	0.042	0.034	0.000	0.007
UF1	2048	0.851	∗0.807	∗0.825	∗0.000	∗0.535
		0.011	0.036	0.017	0.000	0.030
UF1	4096	∗0.757	∗0.806	0.807	∗0.000	∗0.518
		0.091	0.037	0.029	0.000	0.045
UF1	8192	∗0.190	∗0.630	0.815	∗0.000	∗0.510
		0.159	0.062	0.012	0.000	0.041
WFGpos3	1200	∗0.750	∗0.661	∗0.711	∗0.259	0.968
		0.008	0.009	0.008	0.060	0.005
WFGpos3	1400	∗0.749	∗0.597	∗0.688	∗0.246	0.966
		0.006	0.005	0.007	0.059	0.004
WFGpos3	1600	∗0.754	∗0.560	∗0.651	∗0.216	0.966
		0.010	0.006	0.009	0.006	0.004
WFGpos3	1800	∗0.755	∗0.524	∗0.624	∗0.229	0.967
		0.009	0.005	0.009	0.061	0.004

from three aspects: efficiency (i.e., the computational effort that a given algorithm requires to reach the Pareto front), quality of obtained solution sets, and the search behavior of a given algorithm in the objective space. To be specific, the speed metric used in [5], [6] was adopted. It is defined as the number of fitness evaluations needed by algorithms to produce a solution set of expected quality. The expected quality is measured by the hypervolume [26], [31] of the Pareto front of the given problem. In this work, it is set to 90% of the optimal hypervolume as a trade-off between achieving an accurate approximation and the limited computational budget, which becomes particularly relevant in large-scale optimization. The maximum number of fitness evaluations is set to 1.0E+07. For the calculation of the hypervolume in performance comparisons, a slightly worse point than the nadir point is usually used as a reference point in the evolutionary multi-objective optimization community [31]. Thus, we first add 0.1 to each dimension of the nadir point of the given problem. Then, the population is normalized using the ideal point of the problem and the modified nadir point. After the normalization, the hypervolume is calculated using $(1, 1)$ as the reference point for bi-objective minimization problems. The relative hypervolume values (i.e., the ratio of the obtained and the optimal hypervolume value) is used to measure the quality of a given solution set. To study the behavior of the algorithms, solution sets obtained at different evolutionary stages were plotted in the objective space and the corresponding evolutionary curves of the relative hypervolume performance were analyzed.

B. Results and Discussions

The performance of the five evaluated algorithms is shown in Table II. Three representative test problems are shown as examples. As expected, the classical MOEAs could not perform well on large-scale problems. They behaved worse in terms of solution quality as well as efficiency than the large-scale WOF-SMPSO on 19 out of 27 problems examined in this experiment (as the examples DTLZ1 and WFG3pos in Table II). For the rest of the problems (as the example UF1 given in Table II), the evaluated large-scale MOEAs, WOF-SMPSO in particular, unlike their high effectiveness and efficiency on the former 19 problems, could not perform well and were inferior to the classical MOEAs. The reasons behind this inability to scale well in these cases will be further investigated below.

For an in-depth analysis of the reasons behind the inadequate scalability of these classical MOEAs, the effect of the number of variables on their search behavior was investigated. The solution sets obtained are illustrated in Fig. 1. The representative results of SMS-EMOA and NSGA-II are shown as examples.

These results revealed two important phenomena. Firstly, in 10 problems (of ZDT, DTLZ and WFG type), uniformly distributed solutions over the entire Pareto front could be easily obtained by all the algorithms, even for the cases with 4096 variables (as shown in Fig. 1). Although sometimes the obtained solution set is far away from the Pareto front, e.g., the case with 4096 variables shown in Fig. 1, it is found that it has enough diversity as well. This is expected, as these problems tend to require algorithms to have stronger convergence ability only, as shown in many-objective optimization studies [32]. Besides, a closer look at the design of these problems shows that these problems have a very few number of diversity-related variables, e.g., 1 or 2, and no correlation between position and distance functions is considered, which make it relatively easy to find solutions with high diversity. These problems are referred to as convergence-focused problems. Note that the mathematical analysis based on the benchmark functions is only used to help deepen our understanding of the experimental results, rather than directly categorizing the problems.

Secondly, uniformly distributed solutions over the Pareto front could not be obtained for the remaining 17 problems (as the examples UF1 and WFG3pos given in Fig. 1) for most evaluated algorithms. Although the challenge in balancing the convergence and diversity for these problems also exists when the number of variables is small [24], [33], it is found that this phenomenon is even more serious when the number of decision variables increases. The obtained solution sets tended to be more clustered in small parts of the Pareto front as the number of decision variables increased even if the algorithms could approach the Pareto front sometimes. Besides, when the algorithms converged to some parts of the Pareto front, the remaining parts they could find in the subsequent search process were small. These problems (CEC 2009 competition and WFGpos type) thus pose a great challenge to the diversification ability for large-scale multi-objective optimization. A similar diversity loss was also reported for large-scale multi-objective distance minimization problems [34].

Furthermore, another interesting phenomenon was also observed in the above experimental results. It is found that although the loss of diversity can be observed on WFGpos problems for some MOEAs when increasing the number of diversity-related variables, such phenomenon is rarely observed when NSGA-II is adopted (as the example WFG3pos given in Fig. 1). Therefore, a deeper analysis of the design of the examined problems was further carried out. It is found that these problems are designed without considering the correlation between position and distance functions [35], for which the diversity and convergence can be pursued relatively independently. Thus modifying position variables will always result in non-dominated solutions. This might verify why the diversity loss is rarely observed for NSGA-II to some extent, thereby indicating that mechanisms of NSGA-II might be a promising approach for such problems and could shed insight on the categorization of problems, as well as on how the diversification and convergence ability should be prioritized in the choice of algorithms/parameters. In contrast, UF problems are designed quite differently from WFG problems by introducing the correlation between position and distance functions, which has been shown to be an important feature in multi-objective optimization [35]. Considering this difference, the WFGpos problems and UF problem are categorized into two groups, i.e., diversity-type I and II problems.

Consequently, the benchmark problems examined in this experiment can be roughly categorized into three groups according to the above analysis: convergence-focused (ZDT, DTLZ, WFG), diversity-type I (WFGpos), diversity-type II (CEC2009 competition) problems. It is noteworthy that this categorization is derived by considering the behavior of three representative MOEAs from different classes, which may justify well the ability of this categorization to capture problem characteristics. Our observation also clearly shows the difficulty of finding an appropriate balance between the convergence and diversity for large-scale MOPs, which has also been well discussed in the recently proposed LSMOP test suite [36]. Note that these problems may have different characteristics and thus different criteria might result in different categorization. For example, different population sizes might affect algorithm performance [8], [37] and thereby lead to different categorization. In this paper, we mainly consider the experimental observations of existing MOEAs when the number of decision variables increases, and the categorization is based on this indicator and the experimental studies.

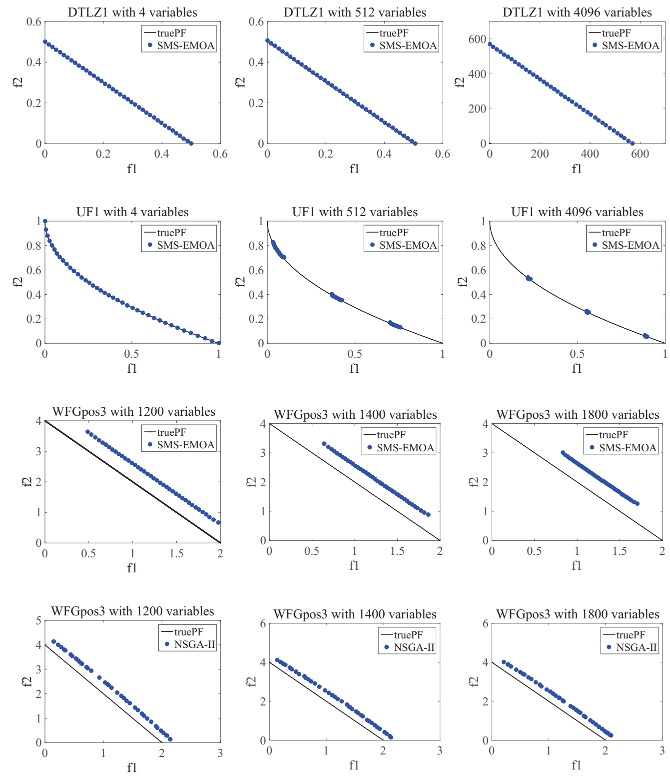

Fig. 1. Solution sets of the runs with the median hypervolume metric values obtained by different algorithms when applied to DTLZ1, WFGpos3 and UF1 with different number of variables

On the basis of the categorization shown above, the performance of the large-scale MOEAs was further investigated. It is found that WOF-SMPSO, which is adapted from the decomposition techniques in large-scale single-objective EAs, has shown a superior performance on 8 out of 10 convergence-focused problems (as the example DTLZ1 given in Table II). This is relatively intuitive since the challenges in achieving good convergence when dealing with these problems can be addressed to some extent by adopting the techniques applied to large-scale single-objective EAs.

However, WOF-SMPSO behaved quite differently when applied to problems with a diversity loss. Although it significantly outperformed other algorithms on all diversity-type I problems (as the example WFG3pos given in Table II), it failed to solve most of the diversity-type II problems (as the example UF1 given in Table II) and even exhibited inferior performance compared to the three classical MOEAs. This might be partly attributed to the relatively complex correlation between position and distance functions in diversity-type II problems, which complicates the identification of a good weighting and would further affect the normal optimization in WOF-SMPSO. On the other hand, it is interesting to find that NSGA-II and WOF-SMPSO, both of which use crowding distance as the selection criterion, do not exhibit an obvious diversity loss when applied to most diversity-type I problems (as the example WFG3pos given in Fig. 1). Thus, combining crowding distance might be helpful when dealing with diversity-type I problems and WOF-SMPSO could be a good option in this scenario. By contrast, for diversity-type II problems, there seems to be a lack of effective techniques in current large-scale multi-objective optimization works, which highlights the need for further attention to these problems.

III. The Proposed DLS-MOEA

Based on the analysis presented in the previous section, popular benchmark problems can be categorized into three groups according to the challenges that tend to be more severe posed by MOEAs when the number of decision variables increases. Considering that convergence-focused and diversity-type I problems can be solved well using techniques employed in the large-scale single-objective optimization context, while existing algorithms degrade severely when dealing with diversity-type II problems, the latter are the focus of this investigation. In this section, a novel MOEA aiming at large-scale MOPs, especially those in the category of diversity-type II, denoted as DLS-MOEA, is presented. Its main objective is to enhance the diversification ability of SMS-EMOA by exploiting a new solution generator, while inheriting the strength of SMS-EMOA in terms of good convergence. While other MOEA frameworks could also be used, SMS-EMOA is adopted as the basic framework in this work since it directly maximizes the hypervolume, and thus could be beneficial to the balance between diversification and convergence. The general idea behind DLS-MOEA is to maintain an external archive and force solutions in the archive to drive the search towards different sub-regions of the Pareto front using a dual local search mechanism.

A. Framework

DLS-MOEA is based on the SMS-EMOA framework and is enhanced by a new solution generator with an external archive. The process starts with a population of randomly created solutions, and an external archive is initialized based on the current population. At each generation, an offspring is created using the new solution generator based on the data from the archive. After merging the offspring with the current population, the hypervolume-based selection is used for the environmental selection of the merged population. The archive is updated by the offspring based on a different rule to encourage diversity. When the search terminates, the non-dominated solutions in the final population are presented as the output.

B. Generating new solutions based on an external archive

The external archive is used as the parent population to generate a set of diverse solutions. Intuitively, the solutions preserved in the archive should also maintain diversity. Thus, the archive is initialized as the better half of the current population, using non-dominated sorting as the first selection criterion and then selecting solutions within the same front at random. The aim of this initialization is to ensure that the archive contains diverse solutions that represent the current non-dominated front, while preserving some promising solutions that would allow to discover other potentially good sub-regions.

Given the current archive, our aim is to generate a set of diverse solutions and then update the archive for the next generation so that different sub-regions of the Pareto front can be searched. Traditional methods often produce a new solution by randomly selecting several parents from the current population and allow the new solution to replace any solution in the population or neighborhood population as long as the fitness of the former is better. While this approach has been shown to perform well when applied to relatively small-scale problems [38], there might be some room for improvements when applied to large-scale MOPs. The reason could be that its actual diversification ability might be inadequate in the large-scale multi-objective optimization since it has no specific mechanism by which the new solutions can be biased towards different parts of the Pareto front. Considering that the performance of the subsequent population update could be highly affected by the newly generated solutions, a new solution generator with a strong diversification ability might be a promising alternative. For this reason, a new solution generator which adopts a different rule in generating and updating new solutions based on the external archive is proposed. More specifically, each member of the archive undergoes a separate search process whereby it could be updated only by a new solution produced using this member as the parent, in case the replacement leads to an improvement in terms of hypervolume indicator. This is done with the goal of forcing each member of the archive to focus locally on one of the sub-regions of the non-dominated front constructed by the current archive. By adopting this approach, the comparatively weaker sub-regions would still be searched, thus preventing limited solution dispersion in the archive.

However, as the mapping between the objective and decision spaces is always unknown in practice, it is not easy to produce a new solution in certain areas in the objective space. Considering this, the local search in the decision space is adopted, under the assumption that probing a relatively limited region within the decision space of a given solution would be more likely to generate a new solution that is within its local area of the objective space. If the assumption is violated, it is possible that the local search become less effective. To alleviate the limitation, the local search is performed only on an external archive. With this strategy, a new solution generated outside its local area may still survive the environmental selection of the population and can therefore benefit the search of other parts of the Pareto front.

Furthermore, to make the local search for each member of the archive more efficiently, the SEE [39], which exhibited a superior performance in large-scale optimization, was adopted here to implement the local search, some parameters of which are modified to adapt to the multi-objective optimization. Note that, in the search process of a specified member, this member is only allowed to be updated by a new solution produced using this member as the parent, and thus the other members can be viewed as constants. Therefore, such process can be viewed as a single-objective local search process on the specified member assisted by a scalar metric named the hypervolume. The main idea of SEE is to decompose a large-scale problem into multiple low-dimensional sub-problems and then optimize each sub-problem with the aid of a meta-model, which aims to predict the local landscape of the decision space by learning two likelihoods that the landscape goes ascending on the smaller side and the landscape goes descending on the larger side, respectively. These two likelihoods are respectively denoted as pl and pr. After being initialized to 1.0, the likelihoods are adjusted adaptively based on the well-known 1/5-rule [40], i.e., the value will be enlarged if the prediction is correct, and be reduced otherwise. The obtained likelihoods are then used for generating a new solution.

To be specific, assuming that the archive denoted as $\boldsymbol{A}$ contains N solutions, i.e., $\boldsymbol{A} = \{\boldsymbol{F}_1, \boldsymbol{F}_2, \cdots, \boldsymbol{F}_N\}$, the specified member is $\boldsymbol{F}_1$ and its corresponding decision vector is $\boldsymbol{x}$ with D decision variables $x_1, \cdots, x_D$, the offspring $\boldsymbol{x}'$ is produced with the aid of the two likelihoods adaptively learned by SEE, as shown in (1):

$$x'_j = \begin{cases} x_j + \Delta x_j, & if\ \Delta x_j > 0 \wedge pr_j < r \\ x_j + \Delta x_j, & if\ \Delta x_j < 0 \wedge pl_j < r \\ x_j, & else. \end{cases} \tag{1}$$

where r is a random number within $[0, 1]$ and Δx_j is randomly generated by the polynomial mutation operator [41]. Then $\boldsymbol{F}'_1$ is obtained from $\boldsymbol{x}'$ and a new archive $\boldsymbol{A}^{(1)} = \{\boldsymbol{F}'_1, \boldsymbol{F}_2, \cdots, \boldsymbol{F}_N\}$ can be obtained accordingly. After that, the hypervolume indicator is used to measure whether $\boldsymbol{F}'_1$ can replace $\boldsymbol{F}_1$ by comparing $HV(\boldsymbol{A}^{(1)})$ and $HV(\boldsymbol{A})$, and thereby the likelihoods of SEE are updated based on the comparison result. It is noteworthy that other indicators can be used as well provided that it can give the rank for $\boldsymbol{F}'_1$ and $\boldsymbol{F}_1$, as SEE only requires to know whether the prediction is correct. The hypervolume is used here since it is an overall performance, and thus is beneficial to the balance between the convergence and diversification. Additionally, instead of using Gaussian and Cauchy mutation operators as in [39], the polynomial mutation is adopted since it has been widely used in the context of multi-objective optimization [41] with effectiveness.

To summarize, given the external archive, the solution generator enters a roll-polling local search phase, in which a new solution based on SEE is created using each member of the archive as the parent before updating the parent by the new solution.

It is noteworthy that this roll-polling local search operation on the external archive can also be viewed as a local search in the objective space. Assuming that the archive denoted as $\boldsymbol{A}$ contains N solutions, i.e., $\boldsymbol{A} = \{\boldsymbol{F}_1, \boldsymbol{F}_2, \cdots, \boldsymbol{F}_N\}$, the local search starts from $\boldsymbol{A}$, and then attempts to find a better archive by incrementally changing a single element of $\boldsymbol{A}$. If the change produces a better archive, an incremental change is made to the new archive. This process is repeated until all the elements have been evaluated. It is noteworthy that each time a single element of $\boldsymbol{A}$ is considered, all the other elements remain unchanged. This process is shown in (2):

$$\boldsymbol{A} = \{\boldsymbol{F}_1, \boldsymbol{F}_2, \cdots, \boldsymbol{F}_N\} \rightarrow \begin{array}{c} \boldsymbol{A}^{(1)} = \{\boldsymbol{F}'_1, \boldsymbol{F}_2, \cdots, \boldsymbol{F}_N\} \\ \cdots \\ \boldsymbol{A}^{(N)} = \{\boldsymbol{F}_1, \boldsymbol{F}_2, \cdots, \boldsymbol{F}'_N\}. \end{array} \tag{2}$$

In this process, hypervolume is used to measure the quality of the archive, as shown in (3):

$$\boldsymbol{A}' = \begin{cases} \boldsymbol{A}, & if\ HV(\boldsymbol{A}) > HV(\boldsymbol{A}^{(i)}) \\ \boldsymbol{A}^{(i)}, & if\ HV(\boldsymbol{A}) \leq HV(\boldsymbol{A}^{(i)}). \end{cases} \tag{3}$$

As a consequence, a better archive $\boldsymbol{A}'$ can be obtained. As the local search can be viewed as being conducted in both the objective and decision spaces, it is termed as dual local search. The resultant DLS-MOEA based on the basic framework of SMS-EMOA can thus be viewed as a memetic method.

C. Detailed Steps of DLS-MOEA

The details of DLS-MOEA are provided in Algorithm 1. Although the new solution generator is capable of generating diverse solutions, its convergence ability is relatively low in comparison to those based on elite population. Hence, when generating new solutions, DLS-MOEA alternates between the new solution generator and the classical solution generator used in SMS-EMOA. In the evolution, a particular solution generator is applied for a fixed number of generations. The algorithm starts by randomly initializing a population of solutions. In every iteration of the main loop, the classical solution generator is first used for generating new solutions (Line 6–7). When executed $maxGen$ times, an external archive is initialized based on the current population, and the parameters to be used in the new solution generator are also initialized (Line 13–14). Next, the new solution generator is used for generating new solutions based on the external archive (Line 18–20). Each time a new solution is generated, it is merged with the current population, and the hypervolume indicator is used for the environmental selection of the merged population. When the iterative optimization is finished, the non-dominated solutions in the final population are returned as the output.

Algorithm 1: DLS-MOEA

Input : The number of generations for each generator $maxGen$, the number of decision variables D, the population size N
Output: An approximation set P

1 Initialize the population P randomly;
2 **while** $stopping\ criterion\ is\ not\ met$ **do**
3 Set the generation counter $g = 0$;
4 **while** $g < maxGen$ **do**
5 **for** $i = 1 : N$ **do**
6 Randomly select p_1, p_2 as the parents from P;
7 Generate q by applying crossover and mutation to p_1, p_2;
8 Update P with q using the hypervolume indicator;
9 **end**
10 Set $g = g + 1$;
11 **end**
12 Set $K = N/2$;
13 Initialize the archive $A = \{F_1, \cdots, F_K\}$ as the better half of P by using the non-dominated sorting and selecting solutions within the same level at random;
14 Initialize the two-dimensional probability vectors $pl_{K\times D}$ and $pr_{K\times D}$, and each element is set to 1.0;
15 Set the generation counter $g = 0$;
16 **while** $g < maxGen$ **do**
17 **for** $i = 1 : K$ **do**
18 Generate F_i' from F_i by using SEE with pl_i and pr_i;
19 Update A by $A^{(i)} = \{F_1, \cdots, F_i', \cdots, F_K\}$ based on (3) using hypervolume indicator;
20 Adjust pl_i and pr_i using the 1/5-rule;
21 Update P with F_i' using hypervolume indicator;
22 **end**
23 Set $g = g + 1$;
24 **end**
25 **end**
26 **return** non-dominated solutions in P;

D. Computational Complexity Analysis

The large number of decision variables of large-scale MOPs increases the computational complexity of MOEAs. As DLS-MOEA is based on the basic framework of SMS-EMOA, in order to analyze the time complexity of DLS-MOEA, it is worthy to first analyze the additional computational cost in each generation introduced by incorporating the new solution generator. Note that there is always only one offspring generated each time the population is updated, regardless of whether this new solution generator is included or not. Therefore, the additional computational cost in each generation comes from two parts: updating the archive with the offspring (which has already been evaluated) (line 19 in Algorithm 1) and adjusting the mutation probabilities by the 1/5-rule (line 20 in Algorithm 1). In the former, the updating is carried out by comparing the offspring with its parent using the hypervolume indicator. As the computational cost of fitness evaluations is often very costly in real-world problems, especially in large-scale problems, this updating time could often be acceptable. Adjusting mutation probabilities contains only simple scalar calculations, which could be omitted compared to the computational cost of fitness evaluation. Given a large-scale MOP with D decision variables and a population size of N, the time complexity of the above two parts are $O(N \log(N))$ and $O(D)$, respectively. Therefore, the computational complexity of the proposed approach is of the same order as that of SMS-EMOA.

IV. Experimental Studies

A. Test Problems and Performance Metrics

The algorithms were evaluated on the diversity-type II problems described in Section II. Each problem involves four instances, with 1024, 2048, 4096 and 8192 variables, respectively. The performance indicators used in the experiments are the relative hypervolume values and the speed metric, as defined in Section II. For all the results, the two-sided Wilcoxon ranksum test [42] at a 0.05 significance level was also conducted to see whether the performance of two algorithms is statistically significantly different.

B. Compared Algorithms

The algorithms compared in this work are CCGDE3 [10], MOEAD/DVA [13], LMEA [14], [43], WOF [17], ReMO [20], NSGA-II [28], MOEA/D [29] and SMS-EMOA [25]. The first five are recent large-scale optimizers that belong to two different groups. To be specific, CCGDE3, MOEA/DVA, LMEA and WOF are based on decomposition techniques for which source code is available online, whereas ReMO employs dimensionality reduction methodology and its source code was obtained from the authors. The remaining algorithms are representative MOEAs from the Pareto-based, decomposition-based and indicator-based classes, which were implemented in the jMetal framework for Java, version 4.5 [44].

The general and algorithm-specific parameter settings of the large-scale optimizers are summarized in Table I and Table III, respectively. They were configured with the settings suggested by their authors. Specifically, for the WOF framework, WOF-SMPSO was included since it exhibited the best performance in the original study. For the same reasons, Re-NSGA-II was included for the ReMO approach.

The proposed DLS-MOEA was configured in the same way as SMS-EMOA, with the exception of implementing a new solution generator, where the polynomial mutation was adopted in SEE. The distribution index of the polynomial mutation, which controls the spread of children solutions around parent solutions, was set to 200.0, aiming to prioritize local search. The number of generations for each solution generator needs to be set for the DLS-MOEA to ensure an appropriate balance between diversity and convergence. Generally, its optimal value is problem-dependent and may vary as the search progresses. To make a fair comparison, one value was used for all the instances, rather than fine-tuning it for each case. In the experiments, it was set to 2.0E+4, as established in the preliminary experiment. All tested algorithms except MOEA/DVA and LMEA were evaluated on each problem instance. MOEA/DVA and LMEA were only tested on problems with 1024 variables because the total number of fitness evaluations used for the analysis before the actual optimization had substantially exceeded the maximal fitness evaluations allowed, i.e., 1.0E+7, for higher dimensionality.

TABLE III
ALGORITHM PARAMETER SETTINGS FOR CCGDE3, MOEA/DVA, LMEA AND DLS-MOEA

CCGDE3	
Species Number	2
Number of generation for each species	1
CR	0.5
F	0.5
MOEA/DVA	
Number of Interaction Analysis	6
Number of Control Property Analysis	20
SBX Crossover η_c (for ZDT, DTLZ and WFG)	15
Crossover Probability	1.0
CR (for CEC 2009 competition)	1.0
F (for CEC 2009 competition)	0.5
Polynomial Mutation η_m	15
Mutation Probability	$1/D^*$
LMEA	
Number of Variable Clustering Analysis	2
Number of Perturbations	4
Number of Interaction Analysis	6
SBX Crossover η_c (for ZDT, DTLZ and WFG)	20
Crossover Probability	1.0
CR (for CEC 2009 competition)	1.0
F (for CEC 2009 competition)	0.5
Polynomial Mutation η_m	20
Mutation Probability	$1/D^*$
DLS-MOEA	
Number of Generations for Each Phase	2.0E+4
SBX Crossover η_c	20
Crossover Probability	0.9
Polynomial Mutation η_m	20
Mutation Probability	$1/D^*$
Polynomial Mutation η_m in SEE	200

*D is the number of decision variables

C. Experimental Results

The performance in terms of the speed and the relative hypervolume values of the peer algorithms—NSGA-II, MOEA/D, SMS-EMOA, Re-NSGA-II, CCGDE, WOF-SMPSO—on the diversity-type II problems are shown in Tables IV–V. The results of MOEA/DVA and LMEA are shown in Table VI. In these tables, the comparison results between the relative hypervolume values yielded by DLS-MOEA and compared algorithms according to the Wilcoxon rank sum test are also presented.

The efficacy of DLS-MOEA can be evaluated from the best performance it has achieved on the examined diversity-type II problems. To be specific, DLS-MOEA not only scaled best on 10 out of 28 instances of the diversity-type II as shown in Table IV, but also obtained the best solutions on 22 instances in terms of the relative hypervolume values according to the summary of the Wilcoxon rank sum test results shown in Table V, which is significantly better than all tested algorithms.

The difficulty of identifying an optimal balance between convergence and diversity is a long-standing issue in the MOEA design, as well as in large-scale multi-objective optimization [34], [45]. Thus, these results indicate that DLS-MOEA not only achieved the best performance on the examined diversity-type II problems, but also exhibited a good balance between diversity and convergence.

To further visualize the experimental results, in Fig. 2, the obtained solution sets in the objective space for the UF1, UF2, UF3 and UF7 problems with 1024 variables are plotted. It can be seen that the solution sets yielded by DLS-MOEA can disperse along the Pareto fronts more widely and evenly than those provided by the other six algorithms. These results suggest that DLS-MOEA can achieve the approximations with both promising diversity and outstanding convergence for these test instances.

The above results also demonstrate the contributions of the new solution generator in DLS-MOEA. We would like to highlight the good performance of the proposed DLS-MOEA in comparison to SMS-EMOA, as the only difference stems from incorporating the new solution generator in the former. Indeed, it performed significantly better than SMS-EMOA on 20 out of 28 instances of the diversity-type II, as shown in Table V. The solution sets yielded by DLS-MOEA and SMS-EMOA shown in Fig. 2 further confirm that the new solution generator is capable of improving the diversification ability. These results indicate that DLS-MOEA not only enhances the diversification ability of SMS-EMOA by exploiting the new solution generator, but also inherits its strength in convergence.

It can be observed from the results that CCGDE3 and Re-NSGA-II have the worst performance on all diversity-type II problems (the only exceptions are NSGA-II and WOF-SMPSO, which performed worse on UF1 with 8192 variables). This outcome could be attributed to the coevolution methods used in CCGDE3 and the random embedding methods used in Re-NSGA-II, which are not suitable for the test instances since these methods aim to reduce the dimensionality of the original problems and are thus more likely to miss the optima.

Although WOF-SMPSO is also a decomposition-based approach, it mitigates this issue to some extent by extending the concept of weighting the variables to multiple objectives and optimizing the weight vectors. Indeed, it achieves a superior performance on problems that only require an algorithm to have strong convergence and diversity-type I problems where no correlation exists between position and distance functions. In addition to the advantages inherited from large-scale single-objective techniques, the good performance of WOF-SMPSO could be partly attributed to the crowding distance scheme, as diversity loss was rarely observed in Section II for NSGA-II (which also adopts crowding distance scheme) when applied to diversity-type I problems. However, it may potentially be trapped in a poor performance when the Pareto optimal sets of a given problem are very complex. The dramatic changes in the performance thus render the WOF-SMPSO unsuitable for solving such problems. On the other hand, by forcing the search towards different sub-regions of the Pareto front, DLS-MOEA is more diversity-oriented. It can also achieve an outstanding convergence, as it adopts a hypervolume indicator-based environmental selection. Thus, DLS-MOEA has an advantage in dealing with large-scale MOPs.

MOEA/DVA and LMEA are large-scale optimizers which aim to identify optimal variable groupings by dividing them into diversity-related and convergence-related variables. The results presented in Table VI indicate that MOEA/DVA can obtain better results on UF4, UF5 and UF6, thus showing

TABLE IV

PERFORMANCE COMPARISONS IN TERMS OF THE SPEED (EFFICIENCY) OF THE ALGORITHMS (MEAN AND STANDARD DEVIATION) WHEN APPLIED TO THE DIVERSITY-TYPE II PROBLEMS WITH DIFFERENT NUMBERS OF DECISION VARIABLES. A DASH '–' INDICATES THAT THE ALGORITHM CANNOT ACHIEVE THE REQUIRED APPROXIMATION FRONT IN 25 RUNS AFTER USING UP THE MAXIMAL FITNESS EVALUATIONS. THE BEST PERFORMANCE VALUES BASED ON THE MEAN AND STANDARD DEVIATION ARE HIGHLIGHTED IN GREY. AN INSTANCE IS SHOWN IN THE TABLE IF THERE EXISTS AT LEAST ONE ALGORITHM THAT CAN ACHIEVE THE REQUIRED APPROXIMATION BEFORE USING UP THE MAXIMAL FITNESS EVALUATIONS. THE PAIR-WISE COMPARISONS BETWEEN DLS-MOEA AND OTHER ALGORITHMS ARE SHOWN IN ♯, ∗ AND §, WHICH INDICATE DLS-MOEA PERFORMED SIGNIFICANTLY WORSE, BETTER AND COMPARATIVELY THAN THE SPECIFIED ALGORITHM, RESPECTIVELY.

Problem	D	NSGA-II	MOEA/D	SMS-EMOA	Re-NSGA-II	CCGDE3	WOF-SMPSO	DLS-MOEA
UF1	1024	∗9.61E+06 1.78E+05	–	–	–	–	–	4.56E+06 1.10E+06
UF1	2048	–	–	–	–	–	–	7.06E+06 1.09E+06
UF2	1024	∗4.17E+06 1.80E+06	∗3.86E+06 9.64E+05	∗2.93E+06 2.27E+06	–	–	–	1.57E+06 9.57E+05
UF2	2048	–	∗7.74E+06 1.95E+06	∗4.69E+06 1.83E+06	–	–	–	1.88E+06 7.50E+05
UF2	4096	–	–	∗6.40E+06 1.52E+06	–	–	–	3.51E+06 7.57E+05
UF2	8192	–	–	–	–	–	–	7.55E+06 1.37E+06
UF3	1024	–	–	∗5.18E+06 1.46E+05	–	–	♯1.43E+04 4.81E+03	1.79E+06 4.48E+04
UF3	2048	–	–	∗9.68E+06 1.65E+05	–	–	♯1.18E+04 3.85E+03	2.78E+06 1.71E+05
UF3	4096	–	–	–	–	–	♯1.02E+04 4.29E+03	5.80E+06 4.36E+05
UF3	8192	–	–	–	–	–	♯9.94E+03 3.34E+03	–
UF7	1024	∗4.55E+06 1.13E+06	–	∗5.82E+06 1.01E+06	–	–	–	2.35E+06 1.21E+06
UF7	2048	∗8.85E+06 1.21E+06	–	–	–	–	–	3.54E+06 2.21E+06
UF7	4096	–	–	–	–	–	–	3.70E+06 1.32E+06
UF7	8192	–	–	–	–	–	–	6.92E+06 1.13E+06

TABLE V

PERFORMANCE COMPARISONS IN TERMS OF RELATIVE HYPERVOLUME VALUES (MEAN AND STANDARD DEVIATION) WHEN THE ALGORITHMS ARE APPLIED TO THE DIVERSITY-TYPE II PROBLEMS WITH 1024 VARIABLES. THE BEST PERFORMANCE BASED ON THE MEAN AND STANDARD DEVIATION IS HIGHLIGHTED IN GREY. THE PAIR-WISE WILCOXON RANK SUM TEST OF DLS-MOEA AGAINST THE COMPARED ALGORITHM IS SUMMARIZED IN TERMS OF THE WIN-TIE-LOSS COUNTS. THE PAIR-WISE COMPARISONS BETWEEN DLS-MOEA AND OTHER ALGORITHMS ARE SHOWN IN ♯, ∗ AND §, WHICH INDICATE DLS-MOEA PERFORMED SIGNIFICANTLY WORSE, BETTER AND COMPARATIVELY THAN THE SPECIFIED ALGORITHM, RESPECTIVELY.

D	Problem	NSGA-II		MOEA/D		SMS-EMOA		Re-NSGA-II		WOF-SMPSO		CCGDE3		DLS-MOEA	
1024	UF1	∗0.876	0.019	∗0.829	0.042	∗0.828	0.034	∗0.000	0.000	∗0.567	0.007	∗0.447	0.110	0.944	0.019
1024	UF2	∗0.912	0.013	∗0.921	0.012	∗0.930	0.017	∗0.761	0.008	∗0.875	0.002	∗0.634	0.016	0.948	0.009
1024	UF3	∗0.881	0.003	∗0.849	0.004	∗0.930	0.004	∗0.376	0.002	∗0.956	0.001	∗0.574	0.007	0.982	0.002
1024	UF4	∗0.683	0.010	∗0.764	0.006	§0.825	0.005	∗0.543	0.006	§0.825	0.015	∗0.613	0.029	0.825	0.006
1024	UF5	§0.527	0.075	∗0.472	0.075	§0.518	0.068	∗0.000	0.000	∗0.002	0.010	∗0.000	0.000	0.538	0.050
1024	UF6	§0.639	0.118	∗0.549	0.141	♯0.670	0.042	∗0.000	0.000	∗0.125	0.120	∗0.000	0.000	0.652	0.083
1024	UF7	∗0.897	0.066	∗0.642	0.165	∗0.809	0.153	∗0.000	0.000	∗0.552	0.047	∗0.137	0.014	0.951	0.137
2048	UF1	∗0.851	0.011	∗0.807	0.036	∗0.825	0.017	∗0.000	0.000	∗0.535	0.030	∗0.395	0.131	0.895	0.029
2048	UF2	∗0.878	0.005	∗0.898	0.006	∗0.918	0.024	∗0.759	0.006	∗0.875	0.002	∗0.284	0.024	0.942	0.011
2048	UF3	∗0.793	0.003	∗0.750	0.006	∗0.903	0.002	∗0.373	0.002	∗0.963	0.001	∗0.511	0.008	0.974	0.004
2048	UF4	∗0.653	0.008	∗0.712	0.005	∗0.808	0.003	∗0.541	0.006	♯0.828	0.012	∗0.519	0.002	0.826	0.005
2048	UF5	§0.436	0.173	∗0.311	0.082	§0.493	0.115	∗0.000	0.000	∗0.001	0.004	∗0.000	0.000	0.477	0.139
2048	UF6	§0.653	0.061	∗0.590	0.118	§0.666	0.041	∗0.000	0.000	∗0.063	0.105	∗0.000	0.000	0.629	0.069
2048	UF7	∗0.874	0.088	∗0.609	0.166	∗0.845	0.106	∗0.000	0.000	∗0.463	0.038	∗0.121	0.010	0.936	0.088
4096	UF1	∗0.757	0.091	∗0.806	0.037	∗0.807	0.029	∗0.000	0.000	∗0.518	0.045	∗0.338	0.114	0.868	0.025
4096	UF2	∗0.869	0.006	∗0.875	0.006	∗0.903	0.020	∗0.761	0.007	∗0.875	0.002	∗0.000	0.000	0.931	0.010
4096	UF3	∗0.774	0.002	∗0.685	0.003	∗0.801	0.004	∗0.371	0.001	♯0.967	0.001	∗0.393	0.009	0.950	0.004
4096	UF4	∗0.599	0.011	∗0.641	0.007	∗0.756	0.005	∗0.543	0.005	♯0.831	0.009	∗0.518	0.001	0.808	0.004
4096	UF5	§0.346	0.200	∗0.135	0.207	§0.331	0.210	∗0.000	0.000	∗0.000	0.000	∗0.000	0.000	0.226	0.192
4096	UF6	∗0.648	0.045	∗0.590	0.124	§0.666	0.061	∗0.000	0.000	∗0.048	0.077	∗0.000	0.000	0.677	0.044
4096	UF7	∗0.738	0.169	∗0.677	0.177	∗0.868	0.028	∗0.000	0.000	∗0.419	0.019	∗0.045	0.016	0.932	0.107
8192	UF1	∗0.190	0.159	∗0.630	0.062	∗0.815	0.012	∗0.000	0.000	∗0.510	0.041	∗0.523	0.236	0.840	0.018
8192	UF2	∗0.817	0.006	∗0.850	0.002	∗0.888	0.015	∗0.762	0.007	∗0.875	0.002	∗0.000	0.000	0.908	0.016
8192	UF3	∗0.724	0.004	∗0.638	0.002	∗0.708	0.003	∗0.370	0.000	♯0.969	0.000	∗0.434	0.005	0.857	0.013
8192	UF4	∗0.561	0.002	∗0.564	0.011	∗0.659	0.006	∗0.541	0.004	♯0.832	0.010	∗0.524	0.001	0.734	0.006
8192	UF5	§0.088	0.057	§0.018	0.114	§0.064	0.102	∗0.000	0.000	∗0.000	0.000	∗0.000	0.000	0.021	0.030
8192	UF6	∗0.658	0.058	∗0.614	0.109	∗0.663	0.079	∗0.000	0.000	∗0.034	0.092	∗0.000	0.000	0.670	0.039
8192	UF7	∗0.280	0.354	∗0.624	0.160	∗0.864	0.027	∗0.000	0.000	∗0.416	0.065	∗0.000	0.000	0.915	0.014
win/tie/loss		22/6/0		27/1/0		20/7/1		28/0/0		22/1/5		28/0/0			

the benefit of variable analysis. However, variable analysis also makes it difficult for MOEA/DVA and LMEA to scale up to MOPs with a larger number of decision variables. To be specific, $O(D^2)$ fitness evaluations, where D is the number of decision variables, are needed for the variable grouping analysis. Thus, this result indicates that, although the variable analysis could be very useful in the optimization process, MOEA/DVA and LMEA might not be good choices for addressing large-scale MOPs. It should be noted that, if off-line analysis could be done in advance or most variables of a given problem are known to be mostly diversity-related, these two algorithms might be good options. Furthermore, LMEA, which also considers the difficulties in dealing with many objectives, could be a good alternative in large-scale many-objective optimization.

TABLE VI
PERFORMANCE COMPARISONS BETWEEN DLS-EMOA, MOEA/DVA AND LMEA ON THE DIVERSITY-TYPE II PROBLEMS WITH 1024 VARIABLES. A DASH '–' INDICATES THAT THE ALGORITHM CANNOT ACHIEVE THE REQUIRED APPROXIMATION FRONT IN 25 RUNS AFTER USING UP THE MAXIMUM FITNESS EVALUATIONS. FOR THE INSTANCES WITH AN '+', THE RELATIVE HYPERVOLUME VALUES ARE REPORTED. FOR THE OTHER INSTANCES, THE SPEED VALUES MEASURED BY THE CONSUMED FITNESS EVALUATIONS ARE REPORTED. THE BETTER PERFORMANCE BASED ON THE MEAN AND STANDARD DEVIATION IS HIGHLIGHTED IN GREY. THE PAIR-WISE COMPARISON BETWEEN DLS-MOEA AND OTHER ALGORITHMS ARE SHOWN IN ♯, ∗ AND §, WHICH INDICATES DLS-MOEA PERFORMED SIGNIFICANTLY WORSE, BETTER AND COMPARATIVELY THAN THE SPECIFIED ALGORITHM, RESPECTIVELY.

	UF1		UF2		UF3		UF4		+UF5		UF6		UF7	
MOEA/DVA	∗9.45E+06	0.00E+00	∗9.45E+06	0.00E+00	∗9.45E+06	0.00E+00	♯9.45E+06	0.00E+00	♯8.72E-01	5.40E-03	♯9.45E+06	0.00E+00	∗9.86E+06	8.18E+0
LMEA	–		∗9.98E+06	2.53E+04	–		–		∗6.00E-06	2.00E-05	–		–	
DLS-MOEA	4.56E+06	1.10E+06	1.57E+06	9.57E+05	1.79E+06	4.48E+04	–		5.38E-01	5.03E-02	–		2.35E+06	1.21E+06

V. CONCLUSION

In this work, an experimental analysis was carried out to investigate the difficulties along with solving large-scale MOPs with an increasing number of decision variables. The results reported here suggest that popular benchmark problems examined in this work can be categorized into three groups, i.e., convergence-focused, diversity-type I and diversity-type II problems. When applied to the convergence-focused problems, algorithms are required to have strong convergence ability, while strong diversification ability is less necessary. Available evidence indicates that these problems can be solved well by using techniques typically applied to large-scale single-objective optimization. When applied to the diversity-type I problems, algorithms are required to have relatively strong diversification ability. Experimental results suggest that combining crowding distance scheme might be helpful when dealing with these problems and WOF-SMPSO could be a good option in this scenario. On the other hand, when dealing with diversity-type II problems with a correlation between position and distance functions, the diversification ability of algorithms becomes a significant challenge, without a marked impact on the convergence. Thus, specific diversification mechanisms are clearly needed for these problems.

Inspired by this observation, a novel MOEA, namely DLS-MOEA, was proposed. It employs a new solution generator with an external archive to force the search towards different sub-regions of the Pareto front. Comprehensive experimental studies were conducted, as a part of which DLS-MOEA was applied to problems with up to 8192 decision variables. Its performance was compared to that of eight state-of-the-art algorithms, confirming its competitiveness in providing a good balance between convergence and diversification as well as its superiority when applied to the examined diversity-type II problems.

In the future, several directions are worthy of further studies: (1) investigating more representative problems to better understand difficulties induced in large-scale multi-objective optimization; (2) constructing more representative benchmark problems to better reflect the characteristics of real-world large-scale problems; (3) developing adaptive problem analysis mechanisms for algorithm selection in large-scale MOEA context; (4) analyzing the impact of population size on large-scale multi-objective evolutionary optimization; (5) applying our algorithm to more large-scale MOPs to better understand its effectiveness; (6) generalizing our idea to other algorithmic frameworks such as MOEA/D and WOF.

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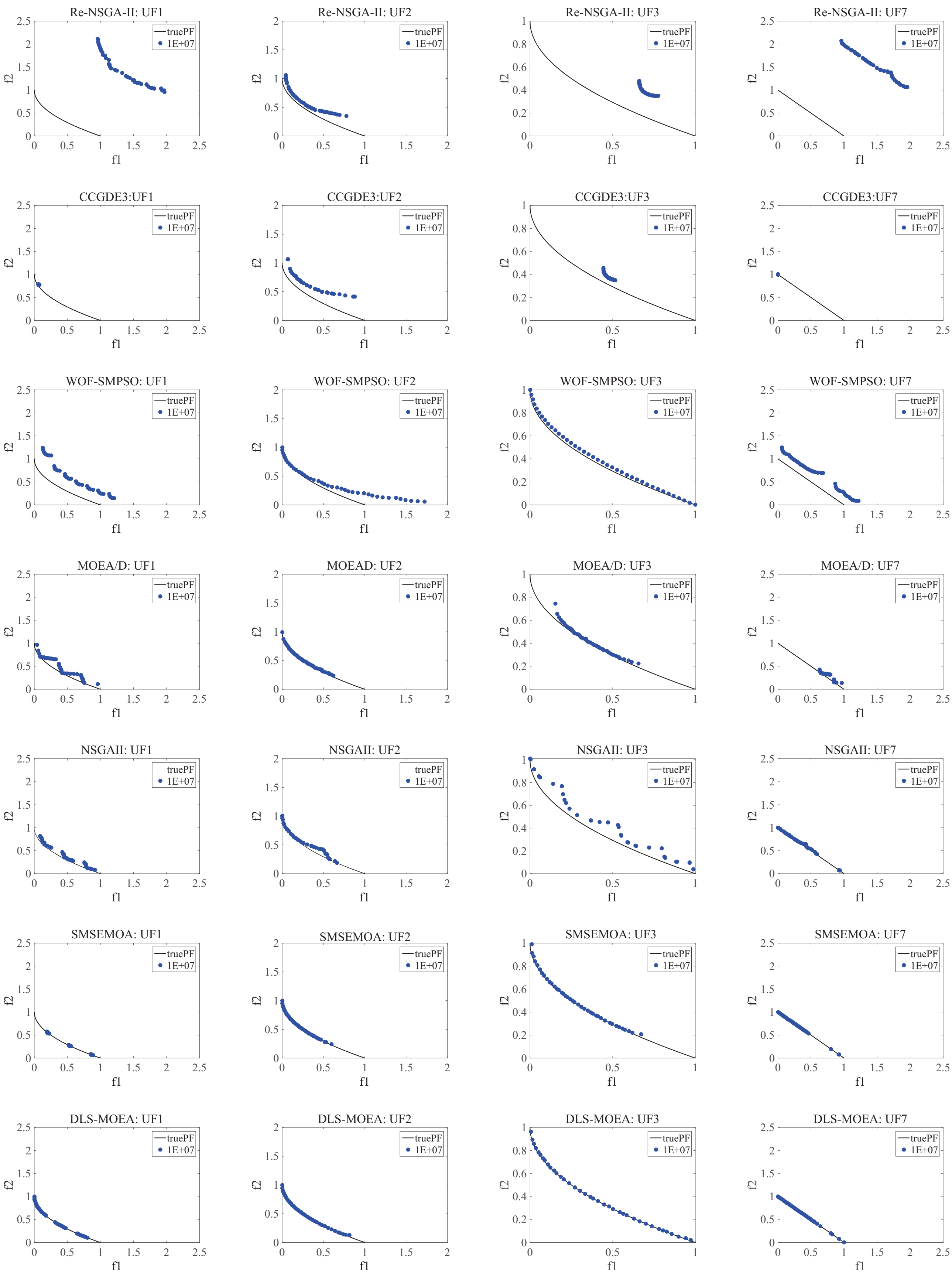

Fig. 2. Solution sets of the runs with the median hypervolume metric values yielded by Re-NSGA-II, CCGDE3, WOF-SMPSO, MOEA/D, NSGA-II, SMS-EMOA and DLS-MOEA when applied to UF1, UF2, UF3 and UF7 with 1024 variables.